not in form (Eq. 8). Mean values of the rate constants given in Table 1, calculated without taking into account of k_1 and k_2 for the last time interval, are: $\bar{k}_1 = 340.8$; $\bar{k}_2 = 415.6$. Most favourable estimators of the rate constants determined by Seinfeld and Gavalas (1970) by the quasilinearization method and equal to: $k_{1,opt} = 347$; $k_{2,opt} = 403$, respectively, differ from the values $\bar{k}_1$ and $\bar{k}_2$ by 2 and 3%, respectively. Thus, the values obtained by means of algebraic Eq. 6 should be considered good initial guesses of parameters.

If it is justified to assume that the measuring error is low then, at a suitably small step h and by applying an approximating formula of high accuracy, the resulting values of rate constants may be considered accurate. For many applications such an approximation may be fairly sufficient. It should be merely remembered that formulas 2 of an order higher than 2 require a constant step.

Because of its simplicity, the suggested method for calculation of the approximate values of model parameters or time dependences of state variables may be of substantial assistance for initial selection of models and for evaluation of the experimental conditions.

NOTATION

a, b = coefficients in formula 2
f = vector of rate of change of concentration
h = integration step size
k = vector of parameters
K = equilibrium constant
m = differential index
n = integration step index
N = number of state variables
R = number of values of t at which data are taken
t = time variable
q = order of formula accuracy
x = state vector
y = observation vector
η = value of t in error formula

LITERATURE CITED

Froment, G. F., "Model Discrimination and Parameter Estimation in Heterogeneous Catalysis," *AIChE J.*, **21**, 1041 (1975).
Himmelblau, D. M., C. R. Jones, and K. B. Bischoff, "Determination of Rate Constants for Complex Kinetics Models," *Ind. Eng. Chem. Fundamentals*, **6**, 539 (1967).
Kittrell, J. R., R. Mezaki, and C. C. Watson, "Estimation of Parameters for Nonlinear Least Squares Analysis," *Ind. Eng. Chem.*, **57**, 18 (1965).
Lapidus, L. and J. H. Seinfeld, *Numerical Solution of Ordinary Differential Equations*, Academic Press, New York (1971).
Pexidr, V., "Evaluation of Nonlinear—Regression Kinetic Data. III. Evaluation of Integral Kinetic Data," *Chem. Prum.*, **24**, 354 (1974).
Seinfeld, J. H., "Nonlinear Estimation Theory," *Ind. Eng. Chem.*, **62**, 32 (1970).
Seinfeld, J. H. and G. R. Gavalas, "Analysis of Kinetic Parameters from Batch and Integral Reaction Experiments," *AIChE J.*, **16**, 644 (1970).
Van den Bosch, B. and L. Hellinckx, "A New Method for the Estimation of Parameters in Differential Equations," *AIChE J.*, **20**, 250 (1974).

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An Explicit Equation for Particle Settling Velocities in Solid-Liquid Systems

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Zanker (1980) has recently presented nomographs for determining particle settling velocities in solid-liquid systems. These nomographs were based on the general correlations developed by Barnea and Mizrahi (1973) and Barnea and Mednick (1975),

$$C_{D_\phi} = \left(0.63 + \frac{4.8}{\sqrt{\mathrm{Re}_\phi}}\right)^2 \tag{1}$$

where

$$C_{D_\phi} = \left(\frac{4d(\rho_d - \rho_c)g}{3\rho_c U_\phi^2}\right)\left(\frac{1-\phi}{1+\phi^{1/3}}\right) \tag{2}$$

and

$$\sqrt{\mathrm{Re}_\phi} = \left(\frac{U_\phi d\rho_c}{\mu_c \exp\,[5\phi/3(1-\phi)]}\right)^{1/2} \tag{3}$$

Settling ϕ equal to zero yields the equations for single particle settling.

In order to avoid an implicit solution for the settling velocity, U_ϕ, from Eqs. 1-3, Zanker presented his nomographs. However, it is shown below that Eqs. 1-3 can be combined to yield an explicit solution for U_ϕ; thus, eliminating the need for either an iterative solution or Zanker's nomographs.

Substitution of Eq. 3 into Eq. 1 and combining the result with Eq. 2 yields

$$U_\phi = a - b\sqrt{U_\phi} \tag{4}$$

where

$$a = \frac{2}{0.63\sqrt{3}}\left[\frac{(\rho_d - \rho_c)gd(1-\phi)}{\rho_c(1-\phi^{1/3})}\right]^{1/2}$$

and

$$b = \frac{4.8}{0.63}\left[\frac{\mu_c \exp\,[5\phi/3(1-\phi)]}{\rho_c d}\right]^{1/2}$$

Equation 4 for U_ϕ can be rewritten in the form of the quadratic

$$U_\phi^2 - (2a + b^2)U_\phi + a^2 = 0 \tag{5}$$

and easily solved to yield the desired result for the settling velocity.

$$U_\phi = c - \sqrt{c^2 - a^2} \tag{6}$$

where

$$c = \frac{1}{2}(2a + b^2) \quad (7)$$

Zanker presented two examples to illustrate the use of his nomographs. In both cases he let

$$\rho_c = 1000\ \mathrm{kg/m^3} \qquad \phi = 0.3$$

$$\rho_d = 2500\ \mathrm{kg/m^3} \qquad \mu_c = 5 \times 10^{-3}\ \mathrm{kg/m/s}$$

For $d = 20 \times 10^{-6}$ m he found $U_\phi = 0.0014$ cm/s and
For $d = 2000 \times 10^{-6}$ m he found $U_\phi = 6.5$ cm/s

Using these values we find $a = 2.036 \times 10^{-2}$, $b = 5.445$, $c = 14.843$ and $U_\phi = 0.001395$ cm/s for the first case and $a = 2.036 \times 10^{-1}$, $b = 5.445 \times 10^{-1}$, $c = 3.518 \times 10^{-1}$ and $U_\phi = 6.49$ cm/s for the second case. These values agree with Zanker's.

It is concluded that Eq. 6 can be used directly to compute particle settling velocities, eliminating the uncertainty associated with nomographs.

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NOTATION

a, b, c = coefficients
C_d = drag coefficient
d = particle diameter, m
g = acceleration due to gravity, $\mathrm{ms^{-2}}$
ρ_c = fluid density, kg $\mathrm{m^{-3}}$
ρ_d = solid particle density, kg $\mathrm{m^{-3}}$
Re_ϕ = Reynold's number
U_ϕ = settling velocity, $\mathrm{ms^{-1}}$
μ = fluid viscosity, kg $\mathrm{m^{-1}\ s^{-1}}$
ϕ = volume fraction of solids

LITERATURE CITED

Barnea, E., and J. Mizrahi, "General Correlation for Fluidization and Sedimentation," *The Chem. Eng. J.*, **5**, No. 2, p. 171 (1973).
Barnea, E., and R. L. Mednick, "Correlations for Minimum Fluidization Velocity," *Trans. Inst. Chem. Engrs.*, **53**, p. 278 (1975).
Zanker, Adam, "Nomographs Determine Settling Velocities for Solid-Liquid Systems," *Chem. Eng.*, p. 147 (May 19, 1980).

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